Received: *May 4, 2001,* Accepted: *June 11, 2001*

Supergravity duals of gauge field theories from $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauged supergravity in five dimensions

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Abstract: We study the SO(4)-symmetric solution of the five-dimensional $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauged $\mathcal{N} = 4$ supergravity theory obtained in [hep-th/0101202]. This solution contains purely magnetic non-abelian and electric abelian fields. It can be interpreted as a reduction of seven-dimensional gauged supergravity on a torus, which comes from type-IIB supergravity on S^3. We also show how to obtain that solution from six-dimensional Romans' theory on a circle. We then up-lift the solution to massless type-IIA supergravity. The dual gauge field theory is twisted and is defined on the worldvolume of a NS-fivebrane wrapped on S^3. Two other spatial directions of the NS-fivebrane are on a torus. In the IR limit it corresponds to a three-dimensional gauge field theory with two supercharges.

Keywords: Brane Dynamics in Gauge Theories, AdS-CFT Correspondance.

JHEP06(2001)025

Contents

1. Introduction

Finding dual field theories of gauged supergravities with solutions involving curved manifolds gives the possibility to explore some aspects of supergravity theories related to twisted field theories through the well-known AdS/CFT duality [1, 2, 3]. Furthermore, this searching certainly provides new examples of the AdS/CFT duality which turn out to be interesting on their own right.

When brane worldvolumes are wrapped on different compact spaces [4]–[10], there are several situations where twisted gauge field theories [11] appear. Particularly, fivebranes and D3-branes wrapped on holomorphic curves were studied [4, 5]. Also, fivebranes [6] and D3-branes [7] wrapped on associative 3-cycles have been investigated, while extensions to M-fivebranes wrapping Kähler 4-cycles, special lagrangian 3-, 4- and 5-cycles, co-associative 4-cycles and Cayley 4-cycles have been systematically studied in reference [8]. In a recent paper, we have studied supergravity solutions describing the flows from AdS_6-type regions to AdS_4 and AdS_3 regions, by considering the large-N limit of D4-branes on 2- and 3-cycles, as well as, wrapped NS-fivebranes [9].

In this paper, we concentrate on a system which, when up-lifted to ten dimensions, can be interpreted as type-IIB NS-fivebranes wrapped on $S^3 \times T^2$. In particular, this S^3 is embedded in a seven-dimensional G_2 holonomy manifold. We consider the decoupling limit of N NS-fivebranes wrapped on $S^3 \times T^2$ [12], keeping the radii fixed. Since the brane worldvolume is curved, in order to define covariantly constant Killing spinors, the resulting field theory on the brane worldvolume will be twisted. In order to describe the flows between the 5+1-dimensional field theory (defined in the NS-fivebrane worldvolume in the UV) and the 2+1-dimensional field theory in

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the IR, we will start with the SO(4)-symmetric solution, obtained by [13] of the five-dimensional SU(2) gauged $\mathcal{N} = 4$ supergravity constructed by Romans [14]. That solution only contains magnetic non-abelian and electric abelian fields. We will show how the five-dimensional supergravity can be obtained from a reduction of the seven-dimensional gauged supergravity theory on a torus. This seven-dimensional theory is obtained by reducing type-IIB supergravity on S^3. The dual twisted field theory is defined on the NS-fivebrane worldvolume wrapped on S^3, whereas the other two spatial directions are wrapped on a torus. In the IR limit it corresponds to a three-dimensional twisted gauge field theory on $\mathbb{R}^1 \times T^2$, with two supercharges. It is worth noting that this theory does not come from an AdS_4-like manifold since its spatial directions does not live in the spatial sector of five-dimensional supergravity, but on the torus in the ten-dimensional theory. In addition, we will see that in the IR the theory is confining. On the other hand, the five-dimensional theory can be viewed as a reduction of six-dimensional gauged supergravity on a circle. This relates it to the Romans' six-dimensional $F(4)$ gauged supergravity theory [15]. Several aspects of this theory as seen from the gauge field theory point of view, including some dual twisted gauge field theories, have been analyzed in reference [9]. We then up-lift the previously mentioned solution to massless type-IIA supergravity on $S^1 \times S^3$.

If we turn off the electric abelian fields, it is also possible to find a solution for the five-dimensional gauged supergravity [13], which indeed is singular. Using the criterion given in reference [4], one can see that the singularity of that solution is *bad*, so that in the IR this solution does not represent a gauge field theory. Therefore, one may say that the electric abelian fields *remove* the singularity. It would be interesting to know whether the non-singular solution with non-vanishing abelian 2-form is related to the rotation of the NS-fivebrane. If it were the case, it would probably be related to the mechanism studied in reference [16], leading to a desingularization by rotation.

This paper is organized as follows. In section 2 we review the basic formalism and set-up of the five-dimensional Romans' theories [14]. The five-dimensional $\mathcal{N} = 4$ AdS supergroup is $\mathrm{SU}(2,2|2)$ whose maximal bosonic supergroup is $\mathrm{SU}(2,2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$. Furthermore, the $\mathrm{SU}(2,2)$ group is isomorphic to $\mathrm{SO}(4,2)$ AdS group in five dimensions. The field content and the lagrangian of this theory will be introduced in the next section, however here we briefly discuss some features of the $\mathcal{N} = 4$ supergravity given in [14]. In fact, the gauge group $\mathrm{SU}(2) \times \mathrm{U}(1)$ generically leads to two coupling constants g_1 and g_2, corresponding to U(1) and SU(2), respectively. In the theories analyzed in [14], four cases are considered depending on the values of g_2 and g_1. In the Romans' paper g_1 is always assumed to be non-zero because in the kinetic term for the self-dual tensor it enters as the factor like $1/g_1$. However, it was pointed out that the limit $g_1 \to 0$ can be taken after some appropriate re-scalings and dualization [17, 13]. Indeed, we will study a solution having only SU(2) gauge symmetry. Hereafter, we will assume $g = g_2$ and $g_1 = 0$, i.e. the U(1) ungauged theory and there are not two 2-index potentials. In section 3, we show how to obtain the five-dimensional $\mathrm{SU}(2)\times\mathrm{U}(1)$ gauged $\mathcal{N}=4$ supergravity from seven-dimensional supergravity, which is obtained by reducing type-IIB supergravity on S^3, on a torus. We also consider the relation to the massless six-dimensional supergravity theory via Kaluza-Klein reduction on a circle. The flow between the 5+1-dimensional gauge theory and the 3-dimensional $\mathcal{N}=1$ SYM theory on a torus is studied in section 4. This flow is driven by the SO(4)-symmetric solution of the Romans' theory in five dimensions obtained in [13]. Discussion will be given in section 5.

2. The Romans' theories in 5 dimensions

In this section we review the five-dimensional $\mathrm{SU}(2)\times\mathrm{U}(1)$ gauged $\mathcal{N}=4$ supergravity constructed by Romans [14], whose conventions we follow. The theory consists of a graviton e_μ^α, three SU(2) gauge potentials A_μ^I, an U(1) gauge potential $\mathcal{A}_\mu$, two 2-index tensor gauge fields $B_{\mu\nu}^\alpha$ which transform as a doublet of U(1), a scalar ϕ, four gravitinos $\psi_{\mu\, i}$ and four gauginos χ_μ. We are interested in the case in which the U(1) coupling constant and two 2-index tensor gauge fields are zero. The bosonic lagrangian of the theory without the two 2-form potentials and U(1) gauge coupling is

$$\begin{aligned} e^{-1}\,\mathcal{L} = &-\frac{1}{4}\,R+\frac{1}{2}\,(\partial^\mu\phi)\,(\partial_\mu\phi)-\frac{1}{4}\,\mathrm{e}^{\frac{4}{\sqrt{6}}\phi}\,F^{I\,\mu\nu}\,F^I_{\mu\nu}-\frac{1}{4}\,\mathrm{e}^{-\frac{8}{\sqrt{6}}\phi}\,\mathcal{A}_{\mu\nu}\,\mathcal{A}^{\mu\nu}+\\ &+\frac{1}{8}\,g^2\,\mathrm{e}^{-\frac{4}{\sqrt{6}}\phi}-\frac{1}{4}\,e^{-1}\,\varepsilon^{\mu\nu\rho\sigma\tau}\,F^I_{\mu\nu}\,F^I_{\rho\sigma}\,\mathcal{A}_\tau\,, \end{aligned} \tag{2.1}$$

where e is the determinant of the vielbein, g is the SU(2) coupling constant and $\varepsilon_{\mu\nu\rho\sigma\tau}$ is a Levi-Civita tensor density. The abelian field strength $\mathcal{A}_{\mu\nu}$ and non-abelian field strength $F^I_{\mu\nu}$ are given by

$$\begin{aligned} \mathcal{F}_{\mu\nu} &\equiv \partial_\mu\mathcal{A}_\nu-\partial_\nu\mathcal{A}_\mu\,,\\ F^I_{\mu\nu} &\equiv \partial_\mu A^I_\nu-\partial_\nu A^I_\mu+g\,\epsilon^{IJK}\,A^J_\mu\,A^K_\nu\,, \end{aligned} \tag{2.2}$$

respectively. The supersymmetry transformations for the gauginos and gravitinos are

$$\delta\chi_a = \frac{1}{\sqrt{2}}\gamma^\mu(\partial_\mu\phi)\epsilon_a+\sqrt{3}T_{ab}\epsilon^b-\frac{1}{2\sqrt{6}}\gamma^{\mu\nu}(H_{\mu\nu\,ab}-\sqrt{2}h_{\mu\nu\,ab})\epsilon^b\,, \tag{2.3}$$

$$\delta\psi_{\mu a} = \mathcal{D}_\mu\epsilon_a+\gamma_\mu T_{ab}\epsilon^b-\frac{1}{6\sqrt{2}}(\gamma_\mu^{\nu\rho}-4\delta_\mu^\nu\gamma^\rho)\left(H_{\nu\rho\,ab}+\frac{1}{\sqrt{2}}h_{\nu\rho\,ab}\right)\epsilon^b\,, \tag{2.4}$$

where T_{ab}, $H_{\mu\nu\,ab}$ and $h_{\mu\nu\,ab}$ are defined as follows

$$T^{ab}\equiv\frac{1}{6\sqrt{2}}\,g\,\mathrm{e}^{-\frac{2}{\sqrt{6}}\phi}\,(\Gamma_{45})^{ab}\,,\quad h^{ab}_{\mu\nu}\equiv\mathrm{e}^{-\frac{4}{\sqrt{6}}\phi}\,\Omega^{ab}\,\mathcal{F}_{\mu\nu}\,,\quad H^{ab}_{\mu\nu}\equiv\mathrm{e}^{\frac{2}{\sqrt{6}}\phi}\,F^I_{\mu\nu}\,(\Gamma_I)^{ab}\,. \tag{2.5}$$

The gauge-covariant derivative $\mathcal{D}_\mu$ acting on the Killing spinor is

$$\mathcal{D}_\mu\,\epsilon_a = \nabla_\mu\,\epsilon_a+\frac{1}{2}\,g\,A^I_\mu\,(\Gamma_{I\,45})_a{}^b\,\epsilon_b\,, \tag{2.6}$$

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with

$$\nabla_\mu \epsilon_a \equiv \left(\partial_\mu + \frac{1}{4}\, \omega_\mu^{\ \alpha\beta}\, \gamma_{\alpha\beta} \right) \epsilon_a \,, \tag{2.7}$$

where $\omega_\mu{}^{\alpha\beta}$ is the spin connection. Indices α, β are tangent space (or flat) indices, while μ, ν are spacetime (or curved) indices. The $\gamma_{\alpha\beta\ldots}$ are the five-dimensional Dirac matrices,

$$\gamma_{\alpha_1\ldots\alpha_n} = \frac{1}{n!}\, \gamma_{[\alpha_1}\, \cdots\, \gamma_{\alpha_n]}\,, \qquad n = 1, \cdots, 5\,.$$

The equations of motion of the lagrangian (2.1) are

$$\begin{aligned} R_{\mu\nu} &= 2\partial_\mu\phi\partial_\nu\phi - 2\mathrm{e}^{-\frac{8}{\sqrt{6}}\phi} \left(\mathcal{F}_\mu{}^\rho \mathcal{F}_{\nu\rho} - \frac{1}{6} g_{\mu\nu} \mathcal{F}_{\rho\sigma}\mathcal{F}^{\rho\sigma} \right) - \\ &\quad -2\mathrm{e}^{\frac{4}{\sqrt{6}}\phi} \left(F^{I\ \rho}_\mu F^I_{\nu\rho} - \frac{1}{6} g_{\mu\nu} F^I_{\rho\sigma} F^{I\ \rho\sigma} \right) + \frac{1}{6} g_{\mu\nu} g^2 \mathrm{e}^{-\frac{4}{\sqrt{6}}\phi}\,, \qquad (2.8) \\ \Box\phi &= -\frac{1}{2\sqrt{6}} g^2 \mathrm{e}^{-\frac{4}{\sqrt{6}}\phi} + \frac{2}{\sqrt{6}} \mathrm{e}^{-\frac{8}{\sqrt{6}}\phi} \mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu} - \frac{1}{\sqrt{6}} \mathrm{e}^{\frac{4}{\sqrt{6}}\phi} F^{I\ \mu\nu} F^I_{\mu\nu}\,, \quad (2.9) \\ \mathcal{D}_\nu \left(\mathrm{e}^{-\frac{8}{\sqrt{6}}\phi} \mathcal{F}^{\nu\mu} \right) &= \frac{1}{4} e \varepsilon^{\mu\nu\rho\sigma\tau} F^I_{\nu\rho} F^I_{\sigma\tau}\,, \qquad (2.10) \\ \mathcal{D}_\nu \left(\mathrm{e}^{\frac{4}{\sqrt{6}}\phi} F^{I\ \nu\mu} \right) &= \frac{1}{2} e \varepsilon^{\mu\nu\rho\sigma\tau} F^I_{\nu\rho} \mathcal{F}_{\sigma\tau}\,. \qquad (2.11) \end{aligned}$$

After some appropriate rescalings of the fields, the lagrangian (2.1) can be written as

$$\begin{aligned} e^{-1}\, \mathcal{L} &= R - \frac{1}{2} \left(\partial^\mu\phi\right) \left(\partial_\mu\phi\right) - \frac{1}{4}\, \mathrm{e}^{-\frac{2}{\sqrt{6}}\phi}\, F^{I\,\mu\nu}\, F^I_{\mu\nu} - \frac{1}{4}\, \mathrm{e}^{\frac{4}{\sqrt{6}}\phi}\, \mathcal{F}_{\mu\nu}\, \mathcal{F}^{\mu\nu} + \\ &\quad + 4\, g^2\, \mathrm{e}^{\frac{2}{\sqrt{6}}\phi} - \frac{1}{8}\, e^{-1}\, \varepsilon^{\mu\nu\rho\sigma\tau}\, F^I_{\rho\sigma}\, F^I_{\tau\kappa}\, \mathcal{F}_\tau\,. \end{aligned} \tag{2.12}$$

The eq. (2.12) is the lagrangian presented in reference [17] with $G_2^{(1)} = G_2^{(2)} = 0$.

3. Obtaining five-dimensional Romans' theory

It was shown in [18] that the five-dimensional SU(2) $\times$ U(1) gauged $\mathcal{N} = 4$ supergravity can be obtained from reduction of type-IIB supergravity on S^5. For our purpose, we are interested in getting the five-dimensional gauged supergravity without the U(1) gauge coupling. Turning off the U(1) gauge coupling can be thought of as taking a singular limit of S^5. In this limit S^5 is deformed to $S^3 \times T^2$. Following this observation, instead of taking the singular limit of the metric and field strength presented in [18], we will show that the SU(2) gauged supergravity in five dimensions can be derived from type IIB on $S^3 \times T^2$.

Let us begin with a subset of the bosonic sector of the ten-dimensional type-IIB supergravity

$$\mathcal{L}_{10} = \hat{R} \hat{*} 1\!\!1 - \frac{1}{2} \hat{*} d\hat{\phi} \wedge d\hat{\phi} - \frac{1}{2}\, \mathrm{e}^{-\hat{\phi}}\, \hat{*}\hat{F}_3 \wedge \hat{F}_3\,. \tag{3.1}$$

The equations of motion of the ten-dimensional theory are

$$
\begin{aligned}
d(\mathrm{e}^{-\hat{\phi}}\hat{*}\hat{F}_3) &= 0\,,\\
d(\hat{*}d\hat{\phi}) &= -\frac{1}{2}\,\mathrm{e}^{-\hat{\phi}}\,\hat{*}\hat{F}_3\wedge\hat{F}_3\,,\\
\hat{R}_{\mu\nu} &= \frac{1}{2}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi}+\frac{1}{2}\mathrm{e}^{-\hat{\phi}}\left[\hat{F}_{\mu\rho\sigma}\hat{F}_\nu{}^{\rho\sigma}-\frac{1}{12}\hat{g}_{\mu\nu}\,\hat{F}_{\rho\sigma\tau}\hat{F}^{\rho\sigma\tau}\right].
\end{aligned}
\tag{3.2}
$$

Following the procedure in [19], we reduce the ten-dimensional theory on S^3 and retain only SU(2) subgroup of a full SO(4) isometry group of S^3. The ansätze for the metric, the scalar and the three form field are

$$
\begin{aligned}
d\hat{s}^2_{10} &= \mathrm{e}^{\frac{3}{\sqrt{10}}\,\tilde{\phi}}\,d\tilde{s}^2_7+\frac{1}{4g^2}\,\mathrm{e}^{-\frac{5}{\sqrt{10}}\,\tilde{\phi}}\sum_{i=1}^{3}(\sigma^i-g\,A^i_1)^2\,,\\
\hat{F}_3 &= \tilde{F}_3-\frac{1}{24g^2}\,\epsilon_{ijk}\tilde{h}^i\wedge\tilde{h}^j\wedge\tilde{h}^k+\frac{1}{4g}\,\tilde{F}^i_2\wedge\tilde{h}^i\,,\\
\hat{\phi} &= \sqrt{10}\,\tilde{\phi}\,,\\
\tilde{h}^i &= \sigma^i-g\,\tilde{A}^i\,,
\end{aligned}
\tag{3.3}
$$

where $\tilde{F}_3=d\tilde{A}_2+\frac{1}{4}\,\tilde{F}^i_2\wedge\tilde{A}^i_1-\frac{1}{24}g\,\epsilon_{ijk}\tilde{A}^i_1\wedge\tilde{A}^j_1\wedge\tilde{A}^k_1$. Substituting the ansätze (3.3) into eq. (3.2), we obtain

$$
\begin{aligned}
d\,\tilde{F}_3 &= \frac{1}{4}\,\tilde{F}^i_2\wedge\tilde{F}^i_2\,,\quad d(\mathrm{e}^{-\frac{4}{\sqrt{10}}\tilde{\phi}}\,\tilde{*}\tilde{F}_3)=0\,,\\
\mathcal{D}(\mathrm{e}^{-\frac{2}{\sqrt{10}}\tilde{\phi}}\,\tilde{*}\tilde{F}^i_2) &= \frac{1}{2}\,\mathrm{e}^{-\frac{4}{\sqrt{10}}\tilde{\phi}}\,\tilde{*}\tilde{F}_3\wedge\tilde{F}^i_2\,,\\
d(\tilde{*}d\tilde{\phi}) &= -\frac{2}{\sqrt{10}}\,\mathrm{e}^{-\frac{4}{\sqrt{10}}\tilde{\phi}}\,\tilde{*}\tilde{F}_3\wedge\tilde{F}_3-\frac{1}{\sqrt{10}}\,\mathrm{e}^{-\frac{2}{\sqrt{10}}\tilde{\phi}}\,\tilde{*}\tilde{F}^i_2\wedge\tilde{F}^i_2-\\
&\quad-\frac{8}{\sqrt{10}}g^2\,\mathrm{e}^{\frac{2}{\sqrt{10}}\tilde{\phi}}\tilde{*}1\!\!1\,,\\
\tilde{R}_{\mu\nu} &= \frac{1}{2}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi}+\frac{1}{4}\,\mathrm{e}^{-\frac{4}{\sqrt{10}}\tilde{\phi}}\left[\tilde{F}_{\mu\rho\sigma}\tilde{F}_\nu{}^{\rho\sigma}-\frac{1}{9}\tilde{g}_{\mu\nu}\,\tilde{F}_{\rho\sigma\tau}\tilde{F}^{\rho\sigma\tau}\right]\\
&\quad\frac{1}{2}\,\mathrm{e}^{-\frac{2}{\sqrt{10}}\tilde{\phi}}\left[\tilde{F}_{\mu\rho}\tilde{F}_\nu{}^{\rho}-\frac{1}{9}\tilde{g}_{\mu\nu}\,\tilde{F}_{\rho\sigma}\tilde{F}^{\rho\sigma}\right]-\frac{2}{3}g^2\,\mathrm{e}^{\frac{2}{\sqrt{10}}\tilde{\phi}}\,\tilde{g}_{\mu\nu}\,.
\end{aligned}
\tag{3.4}
$$

Using odd-dimensional dualization [20] we change 3-form to 4-form

$$
\tilde{F}_4=\mathrm{e}^{-\frac{4}{\sqrt{10}}\tilde{\phi}}\,\tilde{*}\,\tilde{F}_3\,,\ \text{or}\ \ \tilde{F}_3=-\mathrm{e}^{\frac{4}{\sqrt{10}}\tilde{\phi}}\,\tilde{*}\,\tilde{F}_4\,.
\tag{3.5}
$$

In terms of the 4-form field strength, the eqs. (3.4) become

$$
\begin{aligned}
d(\mathrm{e}^{\frac{4}{\sqrt{10}}\tilde{\phi}}\,\tilde{*}\tilde{F}_4) &= \frac{1}{4}\,\tilde{F}^i_2\wedge\tilde{F}^i_2\,,\\
\mathcal{D}(\mathrm{e}^{-\frac{2}{\sqrt{10}}\tilde{\phi}}\,\tilde{*}\tilde{F}^i_2) &= \frac{1}{2}\,\tilde{F}_4\wedge\tilde{F}^i_2\,,\\
d(\tilde{*}d\tilde{\phi}) &= \frac{2}{\sqrt{10}}\,\mathrm{e}^{\frac{4}{\sqrt{10}}\tilde{\phi}}\,\tilde{*}\tilde{F}_4\wedge\tilde{F}_4-\frac{1}{\sqrt{10}}\,\mathrm{e}^{-\frac{2}{\sqrt{10}}\tilde{\phi}}\,\tilde{*}\tilde{F}^i_2\wedge\tilde{F}^i_2-\frac{8}{\sqrt{10}}g^2\,\mathrm{e}^{\frac{2}{\sqrt{10}}\tilde{\phi}}\tilde{*}1\!\!1\,.
\end{aligned}
\tag{3.6}
$$

Eqs. (3.6) together with Einstein's equations (for simplicity, we do not write them down in (3.6)) constitute the equations of motion derived from the lagrangian of SU(2) gauged $\mathcal{N} = 2$ supergravity in seven dimensions without topological mass term [21, 22]

$$\mathcal{L}_7 = \tilde{R}\tilde{*}\mathbb{1} - \frac{1}{2}\,\tilde{*}d\tilde{\phi} \wedge d\tilde{\phi} - \frac{1}{2}\,\mathrm{e}^{\frac{4}{\sqrt{10}}\tilde{\phi}}\,\tilde{*}\tilde{F}_4 \wedge \tilde{F}_4 - \frac{1}{2}\,\mathrm{e}^{-\frac{2}{\sqrt{10}}\tilde{\phi}}\,\tilde{*}\tilde{F}_2^i \wedge \tilde{F}_2^i + \\ +4\,g^2\,\mathrm{e}^{\frac{2}{\sqrt{10}}\tilde{\phi}}\,\tilde{*}\mathbb{1} + \frac{1}{4}\,\tilde{F}_2^i \wedge \tilde{F}_2^i \wedge \tilde{A}_3\,, \tag{3.7}$$

where

$$\tilde{F}_4 = d\tilde{A}_3\,, \qquad \text{and} \qquad \tilde{F}_2^i = d\tilde{A}_1^i + \frac{1}{2}\,\epsilon_{ijk}\,\tilde{A}_1^j \wedge \tilde{A}_1^k. \tag{3.8}$$

The above seven-dimensional gauged supergravity whose lagrangian is eq. (3.7) can also be obtained from an appropriate truncation of a gauged supergravity derived from reducing type-IIA supergravity on S^3 [23]. Having obtained the seven-dimensional gauged supergravity, we reduce it on T^2 following [24, 25, 17]. The ansatz for reduction of the seven-dimensional gauged supergravity on T^2 is

$$\begin{aligned} ds_7^2 &= \mathrm{e}^{\frac{4}{5\sqrt{6}}\phi}\,ds_5^2 + \mathrm{e}^{-\frac{6}{5\sqrt{6}}\phi}\,(dY^2 + dZ^2)\,, \\ \tilde{F}_3 &= \mathcal{F}_2 \wedge dZ \wedge dY\,, \\ \tilde{F}_2^i &= F_2^i\,. \end{aligned} \tag{3.9}$$

The five-dimensional lagrangian obtained from this process is

$$\mathcal{L}_5 = R * \mathbb{1} - \frac{1}{2} * d\phi \wedge d\phi - \frac{1}{2}\,\mathrm{e}^{-\frac{2}{\sqrt{6}}\phi} * F_2^i \wedge F_2^i - \frac{1}{2}\,\mathrm{e}^{\frac{4}{\sqrt{6}}\phi} * \mathcal{F}_2 \wedge \mathcal{F}_2 + \\ +\frac{1}{4}\,F_2^i \wedge F_2^i \wedge \mathcal{A}_1 + 4\,g^2\,\mathrm{e}^{\frac{2}{\sqrt{6}}\phi} * \mathbb{1}\,, \tag{3.10}$$

where

$$\mathcal{F}_2 = d\,\mathcal{A}_1\,, \qquad \text{and} \qquad F_2^i = dA_1^i + \frac{1}{2}\,\epsilon_{ijk}\,A_1^j \wedge A_1^k\,. \tag{3.11}$$

The lagrangian (3.10) is the SU(2) gauged $\mathcal{N} = 4$ supergravity with vanishing U(1) coupling constant and without two 2-form potentials. Eq. (3.10) is written in terms of canonical normalized scalar ϕ and the signature of the space time is mostly plus.

The complete reduction ansatz from 10 to 5 dimensions is

$$\begin{aligned} d\hat{s}_{10}^2 &= \mathrm{e}^{\frac{13}{5\sqrt{6}}\phi}\,ds_5^2 + \mathrm{e}^{\frac{3}{5\sqrt{6}}\phi}\,(dY^2 + dZ^2) + \frac{1}{4g^2}\,\mathrm{e}^{-\frac{3}{\sqrt{6}}\phi}\sum_{i=1}^{3}(\sigma^i - g\,A_1^i)^2\,, \\ \hat{F}_3 &= \mathrm{e}^{\frac{4}{\sqrt{6}}\phi} * \mathcal{F}_2 - \frac{1}{24g^2}\,\epsilon_{ijk}\,h^i \wedge h^j \wedge h^k + \frac{1}{4}\sum_{i=1}^{3} F_2^i \wedge h^i\,, \\ \hat{\phi} &= \sqrt{6}\,\phi\,, \quad \text{and} \quad h^i = \sigma^i - g\,A_1^i\,. \end{aligned} \tag{3.12}$$

The ansatz (3.12) tells us that any solution of the five-dimensional gauged supergravity can be up-lifted to ten dimensions and, this is a solution corresponding to

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the NS-fivebrane. It is not clear that a supergravity obtained from reducing type IIA on S^3 is dual to a theory obtained from reducing type IIB on S^3 in the same sense as T-duality. However, for a particular subset of type-IIA and type-IIB, we will show that type IIA on $S^1 \times S^3$ is equivalent to type IIB on $S^3 \times S^1$. Therefore, any solution of the six-dimensional and the five-dimensional gauged supergravities can be uplifted to either type-IIA or type-IIB supergravities.

The reduction of a subset of type-IIB supergravity on $S^3 \times S^1$ is presented above. It is not hard to see that reducing the seven-dimensional theory whose lagrangian is eq. (3.7) on a circle produces a subset of Romans' theory in six dimensions. On the other hand, the same subset of the Romans' theory in six dimensions was obtained by reducing type IIA on $S^1 \times S^3$ [9], together with the dualization of the 3-form field. The lagrangian and equations of motion of the six-dimensional theory [9] after dualizing the 3-form field are

$$\begin{aligned} \mathcal{L}_6 &= \bar{R} - \frac{1}{2}(\partial\bar{\phi})^2 - \frac{1}{2}\mathrm{e}^{-\frac{1}{\sqrt{2}}\bar{\phi}} \,\bar{*}\bar{F}_2^i \wedge \bar{F}_2^i + 4\,g^2 \mathrm{e}^{\frac{1}{\sqrt{2}}\bar{\phi}}\,\bar{*}1\!\!1 + \frac{1}{4}\,\bar{F}_2^i \wedge \bar{F}_2^i \wedge \bar{A}_2 - \\ &\quad -\frac{1}{2}\mathrm{e}^{\sqrt{2}\bar{\phi}}\bar{F}_3 \wedge \bar{F}_3\,, \\ d(\mathrm{e}^{\sqrt{2}\bar{\phi}}\,\bar{*}\bar{F}_3) &= \frac{1}{4}\sum_{i=1}^{3} \bar{F}_2^i \wedge \bar{F}_2^i\,, \\ \mathcal{D}(\mathrm{e}^{-\frac{1}{\sqrt{2}}\bar{\phi}}\,\bar{*}\bar{F}_2^i) &= \frac{1}{2}\bar{F}_3 \wedge \bar{F}_2^i\,, \\ d(\bar{\phi}\,\bar{*}d\bar{\phi}) &= -\frac{1}{\sqrt{2}}\mathrm{e}^{\sqrt{2}\bar{\phi}}\,\bar{*}\bar{F}_3 \wedge \bar{F}_3 + \frac{1}{2\sqrt{2}}\,\mathrm{e}^{-\frac{1}{\sqrt{2}}\bar{\phi}}\sum_{i=1}^{3}\bar{*}\bar{F}_2^i \wedge \bar{F}_2^i + \frac{4}{\sqrt{2}}\,g^2\,\mathrm{e}^{\frac{1}{\sqrt{2}}\bar{\phi}}\,\bar{*}1\!\!1\,, \end{aligned} \tag{3.13}$$

where $\bar{F}_3 = d\bar{A}_2$ and $\bar{F}_2^i = d\bar{A}_1^i + \frac{1}{2}\epsilon_{ijk}\,\bar{A}_1^j \wedge \bar{A}_1^k$. Reducing the above six-dimensional theory on a circle gives the five-dimensional theory without U(1) gauged coupling. The ansatz of reduction from type-IIA supergravity on $S^1 \times S^3 \times S^1$ is

$$\begin{aligned} d\hat{s}_{10}^2 &= \mathrm{e}^{\frac{7}{8\sqrt{6}}\phi}\,ds_5^2 + \frac{1}{4g^2}\,\mathrm{e}^{-\frac{9}{8\sqrt{6}}\phi}\sum_{i=1}^{3}\left(\sigma^i - g\,A_1^i\right)^2 + \mathrm{e}^{\frac{15}{8\sqrt{6}}\phi}\,dY^2 + \mathrm{e}^{-\frac{9}{8\sqrt{6}}\phi}\,dZ^2\,, \\ \hat{F}_4 &= \left(\mathrm{e}^{\frac{4}{\sqrt{6}}\phi} * \mathcal{F}_2 - \frac{1}{24g^2}\epsilon_{ijk}\,h^i \wedge h^j \wedge h^k + \frac{1}{4g}\,F_2^i \wedge h^i\right) \wedge dY\,, \\ \hat{\phi} &= \frac{3}{4\sqrt{6}}\,\phi\,. \end{aligned} \tag{3.14}$$

4. Duals of 3-dimensional $\mathcal{N} = 1$ SYM theory on a torus

In this section we study the supergravity dual of a 3-dimensional $\mathcal{N} = 1$ SYM theory on a torus. The gravitational system we are dealing with can be understood as follows. Let us consider N type-IIB NS-fivebranes. If the fivebranes were flat, the isometries of this system would be $\mathrm{SO}(1,5) \times \mathrm{SO}(4)$. The first corresponds to the Lorentz group on the flat fivebrane worldvolumes, while the second one is the corresponding rotation group of the S^3 tranverse to the fivebrane directions. Since the NS-fivebranes are not flat but wrapped on a second S^3 (in the five-dimensional Romans' theory), we have the following chain of breaking of the isometries $\mathrm{SO}(1,9) \to \mathrm{SO}(1,5) \times \mathrm{SO}(4) \to \mathrm{SO}(4) \times \mathrm{SO}(4)$. There is also an additional isometry group corresponding to the torus, where the two additional spatial directions of the fivebrane are wrapped. On the other hand, the supergravity solution that we consider here has a global SO(4) symmetry, and its corresponding ansatz for the five-dimensional metric has the $\mathbb{R}^1 \times S^3 \times \mathbb{R}^1$ geometry. The $\mathbb{R}^1$'s correspond to the time and the radial coordinate, respectively. In ten dimensions, the solution has the geometry of the form $(\mathbb{R}^1_0 \times S^3_{1,2,3} \times \mathbb{R}^1_4) \times T^2_{5,6} \times S^3_{7,8,9}$, where the lower indices label the coordinates. Recall from the previous section that the seven-dimensional supergravity is related to the five-dimensional one through a T^2 reduction, whereas the up-lifting to 10-dimensional theory is obtained through an S^3. In the table below, we schematically show the global structure of the ten-dimensional metric. The first five coordinates are arbitrarily chosen to represent the five-dimensional metric for the Romans' theory.

0	1 2 3	4	5 6	7 8 9
$\mathbb{R}^1_0$	$S^3_{1,2,3}$	$\mathbb{R}^1_4$	$T^2_{5,6}$	$S^3_{7,8,9}$

(4.1)

From the table 4.1, one can see that the NS-fivebrane is wrapped on the S^3 (which belongs to the five-dimensional SU(2) gauged supergravity metric ansatz), while its other two spatial directions are wrapped on $T^2_{5,6}$, i.e. the directions 5 and 6.

Now, we focus on the twisting preserving two supercharges. As already mentioned above, there are three spatial directions of the NS-fivebranes wrapped on S^3. Therefore, the supersymmetry will be realized through a twisting. Also notice that the NS-fivebranes have two directions on a torus, so that these are not involved in a twisting. The brane worldvolume is on $\mathbb{R}^1 \times S^3 \times T^2$. The non-trivial part of the spin connection on this worlvolume is the SU(2) connection on the spin bundle of S^3. On the other hand, the normal bundle to the NS-fivebrane in the G_2 manifold is given by $\mathrm{SU}(2) \times \mathrm{SU}(2)$, one of them being the spin bundle of S^3. In this case, the twisting consists in the indentification of the SU(2) group of the spin bundle with one of the factors in the R-symmetry group of the fivebrane, i.e. $\mathrm{SO}(4)_R \to \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. It leads to a diagonal group $\mathrm{SU}(2)_D$, so that it gives a twisted gauge theory. The resulting symmetry group is $\mathrm{SO}(1,2) \times \mathrm{SU}(2)_D \times \mathrm{SU}(2)_R$. In the UV limit the global symmetry is $\mathrm{SO}(1,5) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, so that the four scalars transform as the representation $(\mathbf{1},\mathbf{2},\mathbf{2})$ and there are also 16 supercharges. After the twisting we get 2 fermions (which are the two supercharges of the remaining unbroken supersymmetry) transforming in the $(\mathbf{2},\mathbf{1},\mathbf{1})$ representation of $\mathrm{SO}(1,2) \times \mathrm{SU}(2)_D \times \mathrm{SU}(2)_R$. There are no scalars after twisting, while we get one vector field as it is before the twisting. Therefore, 1/8 of the supersymmetries are preserved, which is related to the fact that the two S^3's, together with the radial coordinate are embedded in a G_2 manifold. In this way, since our IR limit corresponds to set the radial coordinate to be zero, it implies (as we will see) that the S^3 part of the fivebrane will reduce to a point. This is in contrast with the fact that the transverse S^3 and the torus get fixed radii. It shows that when one moves to the IR of the gauge theory (flowing in the radial coordinate on the gravity dual) three of the dimensions become very small and no low energy massless modes are excited on this two-space. Therefore, effectively far in the IR the gauge theory is three-dimensional.

In order to show explicitly how the theory flows to a 3-dimensional SYM theory on a torus, we briefly describe the SO(4)-symmetric solution of the five-dimensional Romans' supergravity presented in [13]. Following that reference, let us consider a static field configuration, invariant under the SO(4) global symmetry group of spatial rotations. As we already mentioned, the metric ansatz has the structure $\mathbb{R} \times S^3 \times \mathbb{R}$ and it can be written as

$$ds_5^2 = e^{2\nu(r)}\, dt^2 - \frac{1}{M(r)}\, dr^2 - r^2\, d\Omega_3^2\,, \tag{4.2}$$

where $d\Omega_3^2$ is the metric on S^3. Notice that here we have adopted the mostly minus signature. In order to define the field configurations on this geometry it is useful to introduce the left-invariant forms σ^i on S^3, such that they satisfy the Maurer-Cartan equation

$$d\sigma^i + \epsilon_{ijk}\, \sigma^j \wedge \sigma^k = 0\,, \tag{4.3}$$

and consequently $d\Omega_3^2 = \sigma^i\, \sigma^i$. We consider the non-abelian gauge potential components written in terms of the left-invariant forms

$$A^i = A^i_\mu\, dx^\mu = [w(r) + 1]\, \sigma^i\,, \tag{4.4}$$

so that they are invariant under the combined action of the SO(4) rotations and the SU(2) gauge transformations. The corresponding field strength is purely magnetic and is given by

$$F^i = dw \wedge \sigma^i + \frac{1}{2}\, [w(r)^2 - 1]\, \epsilon_{ijk}\, \sigma^j \wedge \sigma^k\,, \tag{4.5}$$

while for the abelian gauge potential we consider a purely electric ansatz

$$f(r) = Q(r)\, dt\, \wedge\, dr\,. \tag{4.6}$$

All the rest of functions, i.e. ν, M, w, Q and the dilaton ϕ are only dependent of the radial coordinate r. From the equation of motion of the dilaton the relation

$$\nu(r) = \sqrt{\frac{2}{3}}\, (\phi(r) - \phi_0) \tag{4.7}$$

can be obtained, where ϕ_0 is an integration constant. On the other hand, from the equation of motion of the abelian field, the following result

$$Q(r) = \frac{e^{5\nu(r)}}{\sqrt{M(r)}\, r^3}\,[2\,w(r)^3 - 6\,w(r) + H] \tag{4.8}$$

is derived, where H is an integration constant. The rest of equations of motion with the field configuration and the metric ansatz described above, can be found in reference [13]. The solution must satisfy the equations obtained by setting to zero the supersymmetry transformations for gauginos and gravitinos eqs.(2.3) and (2.4), such that the following first order differential equations are obtained

$$\begin{aligned}
M(r) &= \left(\frac{1}{3}\zeta^2\,V - w\right)^2 + 2\,\zeta^2\,(w^2-1)^2 - \frac{2}{3}\,(w^2-1) + \frac{1}{18\zeta^2}\,,\\
\frac{dw(r)}{d\log r} &= \frac{1}{6\,\zeta^2\,M}\left\{-2\,V\,(w^2-1)\,\zeta^4 + (H-4\,w^3)\,\zeta^2 - w\right\}\,,\\
\frac{d\zeta(r)}{d\log r} &= -\frac{\zeta}{3\,M}\left\{V^2\,\zeta^4 + 12\,\zeta^2\,(w^2-1)^2 - 4\,V\,w\,\zeta^2 + w^2 + 2\right\}\,,
\end{aligned} \tag{4.9}$$

where we have defined $\zeta(r) = \exp[\nu]/r$ and $V(r) = 2\,w(r)^3 - 6\,w(r) + H$. These equations are compatible with the equations of motion derived from the Romans' five dimensional lagrangian given in section 2, and any solution of these first order differential equations preserves two supersymmetries.

Since we are interested in the IR limit, i.e. when $r \to 0$, we obtain the expansions of the functions defining the metric, the magnetic non-abelian and the electric abelian fields for the five-dimensional ansatz. They are

$$\begin{aligned}
w(r) &= 1 - \frac{1}{24}\,r^2 + \cdots,\\
\zeta(r) &= \frac{1}{r} + \frac{7}{288}\,r + \cdots,\\
M(r) &= 1 + \frac{5}{144}\,r^2 + \cdots,
\end{aligned} \tag{4.10}$$

and straightforwardly

$$\nu(r) = \frac{7}{288}\,r^2 + \cdots, \tag{4.11}$$

while for the dilaton we obtain

$$\phi(r) = \phi_0 + \frac{7}{288}\sqrt{\frac{3}{2}}\,r^2 + \cdots. \tag{4.12}$$

In this case we have taken H to be 4. Also, we get

$$Q(r) = \frac{1}{96}\,r + \frac{13}{13824}\,r^3 + \cdots. \tag{4.13}$$

In this way, one can see that in the IR limit the non-abelian gauge potential has a core, while both field strengths, i.e. the abelian and the non-abelian one are of the order r around $r = 0$. Futhermore, the above solution can be up-lifted, following the ansätze presented in the section 3, to either type-IIA or type-IIB theories. In these two cases, the IR limit turns out to be the same.

Up-lifting to type-IIB theory. Firstly, we consider the case when the solution is up-lifted to type-IIB supergravity. From eq. (3.12) the 10-dimensional metric is

$$\begin{aligned} d\hat{s}_{10}^2 &= -\mathrm{e}^{\frac{13}{5\sqrt{6}}\phi}\left(\mathrm{e}^{2\nu(r)}\,dt^2 - \frac{1}{M(r)}\,dr^2 - r^2\,d\Omega_3^2\right) + \mathrm{e}^{\frac{3}{5\sqrt{6}}\phi}\,(dY^2 + dZ^2) + \\ &\quad + \frac{1}{4g^2}\,\mathrm{e}^{-\frac{3}{\sqrt{6}}\phi}\sum_{i=1}^{3}(\sigma^i - g\,A_1^i)^2\,, \\ \hat{\phi} &= \sqrt{6}\,\phi\,. \end{aligned} \tag{4.14}$$

Therefore, using the previously calculated IR expansion we can obtain the radii of the different manifolds. Thus, for the S^3 involving the coordinates 1, 2, and 3, the radius is given by

$$R_{1,2,3}^2 = \mathrm{e}^{13\phi_0/5\sqrt{6}}\,r^2 + \mathcal{O}(r^4)\,, \tag{4.15}$$

so that we can see how the radius of the S^3 in the five-dimensional metric ansatz shrinks to zero in the IR. On the other hand, the radii of $T_{5,6}^2$, $S_{7,8,9}^3$ remain finite as we can see as follows

$$\begin{aligned} R_T^2 &= \mathrm{e}^{\frac{\sqrt{3}}{5\sqrt{2}}\phi_0}\left(1 - \frac{7}{960}\,r^2 + \mathcal{O}(r^3)\right)\,, \\ R_{7,8,9}^2 &= \frac{1}{4g^2}\mathrm{e}^{-\frac{3}{\sqrt{6}}\phi_0}\left(1 - \frac{21}{576}\,r^2 + \mathcal{O}(r^3)\right)\,. \end{aligned} \tag{4.16}$$

Without loss of generality, we can set ϕ_0 to zero. It is obvious from eqs. (4.15) and (4.16) that in the limit $r \to 0$, R_T and $R_{7,8,9}$ remain finite, while $R_{1,2,3} \to 0$. Since the type-IIB NS-fivebrane is wrapped on $S_{1,2,3}^3$, T^2, and in the IR limit $S_{1,2,3}^3$ effectively reduces to a point, in this limit we obtain a twisted gauge field theory defined on the torus.

Up-lifting to type-IIA theory. Now, we consider the metric given in eq. (3.14)

$$\begin{aligned} d\hat{s}_{10}^2 &= -\mathrm{e}^{\frac{7}{8\sqrt{6}}\phi}\left(\mathrm{e}^{2\nu(r)}\,dt^2 - \frac{1}{M(r)}\,dr^2 - r^2\,d\Omega_3^2\right) + \\ &\quad + \frac{1}{4g^2}\,\mathrm{e}^{-\frac{9}{8\sqrt{6}}\phi}\sum_{i=1}^{3}\left(\sigma^i - g\,A_1^i\right)^2 + \mathrm{e}^{-\frac{9}{8\sqrt{6}}\phi}\,dZ^2 + \mathrm{e}^{\frac{15}{8\sqrt{6}}\phi}\,dY^2\,, \\ \hat{\phi} &= \frac{3}{4\sqrt{6}}\,\phi\,. \end{aligned} \tag{4.17}$$

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Therefore, as we did for the type-IIB case, we can obtain the radius

$$R_{1,2,3}^2 = \mathrm{e}^{\frac{7}{8\sqrt{6}}\phi_0}\, r^2 + \mathcal{O}(r^4)\,, \tag{4.18}$$

which shrinks to zero in the IR limit. In addition, the radii of S_5^1, $S_{6,7,8}^3$ and S_9^1 are finite

$$\begin{aligned} R_5^2 &= \mathrm{e}^{\frac{15}{8\sqrt{6}}\phi_0}\left(1 + \frac{105}{4608}\, r^2 + \mathcal{O}(r^3)\right)\,, \\ R_{6,7,8}^2 &= \frac{1}{4g^2}\, \mathrm{e}^{-\frac{9}{8\sqrt{6}}\phi_0}\left(1 - \frac{27}{4608}\, r^2 + \mathcal{O}(r^3)\right)\,, \\ R_9^2 &= \mathrm{e}^{-\frac{9}{8\sqrt{6}}\phi_0}\left(1 - \frac{27}{4608}\, r^2 + \mathcal{O}(r^3)\right)\,. \end{aligned} \tag{4.19}$$

Again, by considering $\phi_0 = 0$, in the IR, the radii $R_5 = R_9$ and $R_{6,7,8} = 1/(2g)$, while $R_{1,2,3} \to 0$. Since the type-IIA NS-fivebrane is wrapped on $S_{1,2,3}^3$, S_5^1 and S_9^1, and in the IR limit $S_{1,2,3}^3$ effectively shrinks to a point as in the type-IIB case, we get the same geometric reduction as in the previous case. Note that this can be obtained when $\phi_0 = 0$, so that the radii of the torus (in type-IIB case) and the two S^1's (in type-IIA case) are exactly the same.

In addition, in both cases one can use the criterion for confinement given in references [26, 27], in order to show that the corresponding static potential is confining.

The singular SO(4)**-symmetric solution.** A solution with no electric abelian fields can be obtained by setting H to zero. It implies that w, V and also Q are zeros, as we expected since no electric field are excited. In this way, the first order differential equations (4.9) can be easily integrated, yielding the relation

$$r = r_0\, \frac{\mathrm{e}^{1/24\zeta^2}}{\sqrt{\zeta}}\,, \tag{4.20}$$

where r_0 is an integration constant. The metric is given by

$$ds_5^2 = r_0^2\, \mathrm{e}^{1/(12\zeta^2)}\left(\zeta\, dt^2 - \frac{1}{8\,\zeta^5}\, d\zeta^2 - \frac{1}{\zeta}\, d\Omega_3^2\right)\,. \tag{4.21}$$

In adition, for the dilaton we have the following relation

$$\mathrm{e}^{\sqrt{2}\phi/\sqrt{3}} = r_0\, \mathrm{e}^{1/24\zeta^2}\, \sqrt{\zeta}\,. \tag{4.22}$$

Using the criterion of reference [4] it is straightforward to see that the IR singularity is not acceptable, both in type-IIA and type-IIB theories.

5. Discussion

From the point of view of the supergravity theories, the SO(4)-symmetric solution of the Romans' five-dimensional theory can be up-lifted to either type-IIB or type-IIA supergravities. These are obtained through the up-lifting to seven and six-dimensional supergravities, respectively. It means that the 10-dimensional system consists of NS fivebranes, either type IIB or IIA. On the other hand, in our previous paper [9], we constructed a non-abelian solution that is identical to the solution obtained in references [28, 29]. It has been interpreted in [5] as a wrapped NS-fivebrane. In this case, it was a gravity dual of a theory very similar to $\mathcal{N} = 1$ super Yang-Mills. In addition, in [9] we interpreted the massless solution of the Romans' six-dimensional theory as the same NS-fivebrane with a compactified direction. In such a situation, the IR theory was $\mathcal{N} = 2$ SYM in three dimensions. In these cases, the fact that their actions in the string frame are similar, for the massless six-dimensional supergravity and the corresponding seven-dimensional one with vanishing topological mass, indicates that those are the same system. We can see a similar issue in the five-dimensional supergravity studied here, since again the 10-dimensional system involves NS-fivebranes. Thus, from the analysis in the present paper, we conclude that this is the natural extension of the six and seven-dimensional results to five dimensions. In fact, in the IR limit this case corresponds to $\mathcal{N} = 1$ super Yang-Mills theory on a torus, which is confining. This IR theory is interesting on its own. Although many aspects of three-dimensional super Yang-Mills theories have been considered [30, 31, 32], some aspects of three-dimensional $\mathcal{N} = 1$ super Yang-Mills theory on a torus are still poorly understood. Therefore, the results obtained here can be an interesting motivation for further studies since we have presented a gravity dual of $\mathcal{N} = 1$ super Yang-Mills theory on a torus.

We remark that the singular solution obtained in [13] is produced when the electric abelian fields are turn off. This solution has a singular g_{00} even when it is considered in the ten-dimensional theory. This means that this solution does not represent any gauge field theory in the IR. We think that it would be interesting to know if the presence of electric abelian fields is related to a rotation of the fivebranes, leading to a desingularization of the solution. Although, we think that this point deserves further investigation, we can discuss here a little about this mechanism. The issue of the resolution of the singularity can be understood as follows. We recall one of the cases studied in [9], which has been interpreted as the gravity dual of the three-dimensional $\mathcal{N} = 2$ super Yang-Mills theory. Actually, this solution is related to the one given in [5], and it represents a smeared NS-fivebrane on S^2 after T-duality. What is worth stressing is that the resolution of the singularity in this case was produced by the excitation of non-abelian fields. In the case of the metric of eq. (4.2), as we have seen, in order to obtain a non-singular metric it is necessary to turn on the electric abelian fields. We also have to recall that for that particular case one forced the metric on the S^3 to be r^2. We think that this fact induces the singularity, so that the non-abelian fields are not enough in order to prevent it, as in the cases of [5, 9]. It would be interesting to see what happens if instead of r^2 we write a more general function of r. It would probably render a similar situation as in [5, 9], i.e. the resolution of the singularity with only non-abelian fields.

Acknowledgments

We are grateful to Carlos Núñez for collaborating in the early stages of this project and for illuminating discussions. We thank Amihay Hanany, Inyong Park, Christopher Pope and Matthew Strassler for helpful discussions. M.S. would like to thank Ian Kogan and Fidel Schaposnik for correspondence. The work of M.S. is supported in part by funds provided by the U.S. Department of Energy under cooperative research agreement #DF-FC02-94ER40818, CONICET of Argentina, Fundación Antorchas of Argentina and The British Council.

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